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Applications: Systems of Differential Equations and Dynamic Systems

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Abstract: **The study dealt with definitions of types of differential equations, the definition of the system of linear differential equations, the dynamic system and the solutions of some types of differential equations .The study concerned with the types of applications of linear and nonlinear differential equations systems. The study focused on some of the dynamic systems and stability (stability) in the dynamic systems and solutions Jacoppin equations for the values of self-propelled and self-propelled vectors (Jacobin: Eigenvalue and Eigenvector). The study focused on the applications of dynamic systems including predator-prey system (Predator - prey system) and epidemic (Epidemic) and the balance in the ecosystem and other applications.**

Keywords: **Differential equations, dynamic systems, applications, linear and nonlinear, Predotor-prey.**

1. INTRODUCTION

Differential equations date back to the mid-seventeenth century, when calculus was discovered independently by Newton (c. 1665) and Leibniz (c. 1684). Modern mathematical physics essentially started with Newton's Principia (published in 1687) in which he not only developed the calculus but also presented his three fundamental laws of motion that have made the mathematical modeling of physical phenomena possible. [1]

2. THE SIGINFICANCE OF THE STUDY

The importance of the study is to give a simple description of the ecosystem and system of differential equations which enable the both reader and teacher to understand the linear and dynamic systems and the ease of solving them, and to know the environmental competition between the elements of the system and its effects on the surrounding elements. The study proves to the recipient that the extinction of each element in the system, environmental identifies other elements of extinction. With reference to the previous studies concluded that - the ideal ecosystem is a security system in which the interdependence of elements.

Statement of the Problem:

The study raises the following question: Are the partial differential equations represent basis for solutions of systems differential equations and fit the solutions of dynamic systems ?

Objectives of the Study:

The purpose of this study is to investigate whether partial differential equations have a role in the solution and applications of differential equations and dynamic systems

To knowcompetitions in the dynamic system of the system members and the spread of infection and control to reduce the impact of the spread of the disease among members of the system.

3. DIFFERENTIAL EQUATION

Definition of differential equations:

Suppose that Y function in the variable X and $Y^{\backslash}$, $Y^{\backslash\backslash}$, …, $Y^{(n)}$ differential derivatives of the first –second order and n, variable y for me X any relationship between X,Y and one previous differential derivatives called differential equation ordinary. However, if the Y variable function and its derivatives partly it is called in more than a partial differential equation and now we mention some examples of different regular and partial differential equations.

$$\frac{d^2y}{dx^2} + w^2y = 0 \rightarrow E.q1$$

$$\left[\frac{d^3y}{dx^3}\right]^2 - 2\left[\frac{dy}{dx}\right]^4 + 2yZ = 5 \rightarrow E.q2$$

Example(1)

Solving the differential equation.

$$\frac{dy}{dx} = (x+y)^2$$

Solution:

$$x + y = z$$

$$\because \frac{dy}{dx} + 1 = \frac{dz}{dx}$$

$$\because \frac{dy}{dx} = \frac{dz}{\partial x} - 1$$

$$\because \frac{dz}{dx} - 1 = z^2 \Rightarrow \int \frac{dz}{z^2+1} - 1 = \int dx + c$$

$$\therefore x + c = tan^{-1}Z \therefore x + c = tan^{-1}(x+y)$$

General solution:

General solution - the solution to the first-order separable differential equation$A(x)dx + B(y)dy = 0 \rightarrow E.q3$

Is $\int \quad A(x)dx + \int \quad B(y)dy = C \rightarrow E.q4$

Where (C) represents an arbitrary constant

The order differential equations:

Order differential equation is derived order higher in the differential equation and the degree of the equation is the Wallace uniforms lifted him higher differential coefficient fixed for the order of the equation.

$$\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2} = \frac{d^2y}{dx^2} \quad \rightarrow Eq5$$

In this Equation is a second - order and second degree.

Linear differential equations:

An n, - order liner differential equation has the form

$$b_n(x)y^n + b_{n-1}(x)y^{(n-1)} + \ldots + b_2(x)y^{\backslash\backslash} + b_1(x)y^{\backslash} + b_0(x)y \quad = g(x) \qquad Eq6$$

Where $g(x)$ and the coefficients $b_j(x)$ $(j = 0,1,2,\ldots,n)$depend solely on the variable x. In other words, they do not depend on y derivative of y.

If$g(x) = 0$,then Eq 6is homogeneous, if not Eq 6is non homogeneous. A linear differential equation has constant coefficients if all the coefficients $b_j(x)$in $[Eq\ 6]$ are constants if one or more of these coefficients is not constant $[Eq\ 6]$ has vas variable coefficient**.**

Theorem:

The nth - order linear homogeneous differential equation $L(y) = 0$ always has n, linerly independent if $y_1(x), y_2(x), \ldots, y_n(x)$ represent these solutions. Then the general solution of$L(y) = 0$ is

$$y \quad (n) = C_1Y_1(x) + C_2Y_2(x) + \ldots + C_nY_n(x) \rightarrow E.q7$$

Where $C_1, C_2, \ldots, C_n$ denote arbitrary constants.

linear system of differential equation:

An$n \times n$ first order linear differential system is the equation.

$$\dot{x}(t) = A(t)X(t) + b(t) \text{——} (8)$$

Where $n \times n$ coefficient matrix A the source $n - vectorb$, and the unknown n-vector x are given in components by .

$$\mathbf{A(t)} = \begin{bmatrix} \mathbf{a_{11}(t)} & \cdots & \mathbf{a_{n1}(t)} \\ \vdots & & \vdots \\ \mathbf{a_{1n}(t)} & \cdots & \mathbf{a_{nn}(t)} \end{bmatrix} \mathbf{b(t)} = \begin{bmatrix} \mathbf{b_1(t)} \\ \vdots \\ \mathbf{b_n(t)} \end{bmatrix} \mathbf{x(t)} \begin{bmatrix} \mathbf{x_1(t)} \\ \vdots \\ \mathbf{x_n(t)} \end{bmatrix}$$

The system (2.1) is called homogeneous if the source vector b=0 of constant coefficients if the matrix A is constant, and diagonalizable if the matrix A is diagonalizable.

Example (2)

Find the coefficient matrix the source vector and the unknown vector for the (2 x 2) linear systems.

$$\mathbf{x_1^1 = a_{11}(t)x_1 + a_{12}(t)x_2 + g_1(t)}$$

$$\mathbf{x_2^1 = a_{21}(t)x_1 + a_{22}(t)x_2 + g_2(t)}$$

Solution:

The coefficient matrix A the source vector b, and the unknown vector x are

$$\mathbf{A(t)} = \begin{bmatrix} \mathbf{a_{11}(t)} & \mathbf{a_{12}(t)} \\ \mathbf{a_{21}(t)} & \mathbf{a_{22}(t)} \end{bmatrix} \mathbf{b(t)} = \begin{bmatrix} \mathbf{g_1(t)} \\ \mathbf{g_2(t)} \end{bmatrix} \mathbf{x(t)} \begin{bmatrix} \mathbf{x_1(t)} \\ \mathbf{x_2(t)} \end{bmatrix}$$

Linear Continuous-time Dynamical Systems:

Definition:

The dynamic of a linear dynamical system are described by a linear differential equation. To describe such a system is to take a matrix of real numbers such a system we take a matrix of real numbers which will represent the dynamics and a vector of real that is the initial point. Here is to use the classical definition and notations.

(linear continuous-time dynamical system) given a matrix $A \in R^{nXn}$ and a vector $X_0 \in R^n$. We define X as the solution of the following Cauchy problem

$$\begin{cases} \mathbf{X` = AX} \\ \mathbf{X_{(0)} = X_0} \end{cases}$$

X is called trajectory of the system

Stability of the Dynamic of System:

Definition of stability

Let us consider a generic dynamic systems (linear / nonlinear, time varying, stationary) described by

$$\mathbf{\acute{x}(t) = f(x(t), u(t), t) \rightarrow E.q9}$$

$$\mathbf{y(t) = g(x(t), u(t), t) \rightarrow E.q10}$$

Assume that $\mathrm{U.U_fX.Y}$are normed factor spaces (equipped with a norm)

Given the initial time instant t_0 the initial state$(\mathrm{xt_0})$and the input function $u_{(.)}$ let consider the reference motion.

$$\mathbf{\bar{x}(t) = \varphi(t, t_0, \bar{x}(t_0), \bar{u}_{(.)})t \geq t_0}$$

It is i interested to study two types of perturbations

1- Variation of motion due to a perturbation of the initial state

$$\delta x_1(t) = \varphi(t, t_0, \bar{x}(t_0) + \delta(t_0), \bar{u}_{(.)}) - \bar{x}(t), t \geq t_0$$

$$\text{differencebetweenmotions}$$

2- Variation of motion due to perturbation of the input

$$\delta x_2(t) = \varphi(t, t_0, \bar{x}(t_0) + \bar{u}(.), \delta\bar{u}_{(.)}) - \bar{x}(t), t \geq t_0$$

Differencebetweenmotions

Predator – Prey System:

Definition

The basic idea with the predator-prey system is that there are two populations one of which preys upon the other. Using a real-life example in the presence of food rabbits breed new rabbits and the rabbit population grows as the rabbit population grows. There is more food for foxes to eat. This causes the fox population to grow. Eventually, there are so many foxes that the rabbit population begins to decline in numbers this results in having many foxes die due starvation and hardship when the fox population diminishes there is little to restrain the rabbit population follows and the cycle repeats itself.

If we identify the rabbit population with the letter X and the fox population with the letter Y we can express this Predator-Prey relationship as in equations (eq3.1and eq3.2)

$$\frac{dx}{dt} = ax - bxy - mx^2 \text{———Eq11}$$

$$\frac{dy}{dt} = cxy - ey - ny^2 \text{———Eq12}$$

Where (a, b, m, c, e and n)are constant parameters. These two equations form an interdependent system of two first-order nonlinear differential equation.

They are nonlinear because of the interactive and power terms $(\text{e.g.}\, xy, x^2, \text{and} y^2)$

In both equations which means that appear in both equations they are interdependent because the variables x and y appear in both equations which means that $\frac{dy}{dt}$depends on both classic predator-prey equations of Lotka (1925)and volterra (1930,1931) less general bersions of these eqquations set parameters m and n to zero.

The predator-prey model is often introduced without the crowding and resource limitation terms $(mx^2 \text{and} ny^2)$ found in equations $(eq11 and eq12)$ to show the population interactions work by themselves. One way to represent the interactions between these two variables is with a time series plot. As is done in figure (5.1). In this figure the scale of the vertical axis is

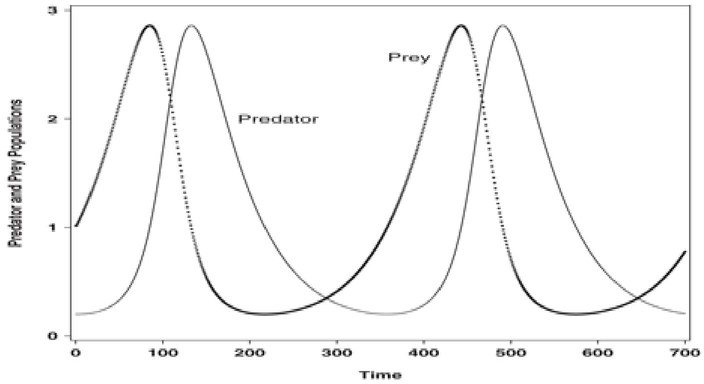

Fig (1) Time series plot of predator-prey model without resource limitations.

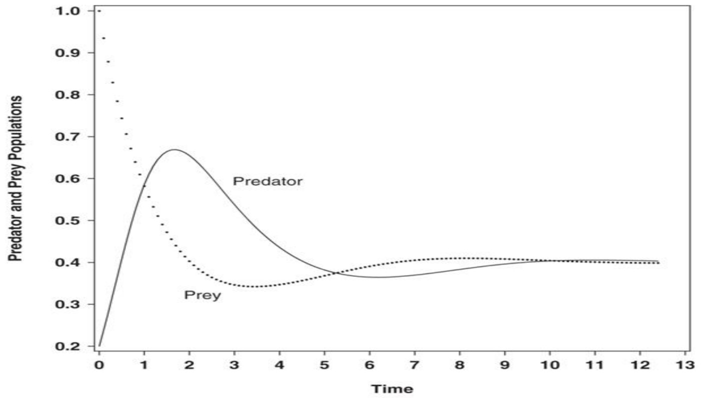

Fig (2) series plot of predator-prey model with resource limitations.

Epidemic:

Modeling epidemics with differential equation

Definition:

The basic model: The population is fixed so $S + I + R = 1$

The disease spreads through interaction of susceptible and infected. We assume that only a fraction of this interaction causes the disease to pass from an individual (***I***) to a susceptible individual (***S***) so the rate of change of ***S*** is proportional to the product of ***S*** and ***I***. we assume that the individuals recover at a rate of β so the period of infection is $\frac{1}{\beta}$ days. The only way a person can leave the infected group is to become infected. The only way a person can leave the infected group is to become recovered. Once a person is recovered the person is no longer susceptible and is immune. Age, Sex, race and social status do not affect the probability of a person being affected. There is no inherited immunity at this time. The people of the population mix homogeneously based on the above assumptions the differential equations governing the disease can be modeled as

$$\frac{ds}{dt} = -\propto SI$$
$$\frac{dI}{dt} = -\propto SI - \beta I$$
$$\frac{dR}{dt} = \beta \text{——} E.q13$$

Remark

Since the total population is assumed to be constant. The third equation can be derived from the first two. Basically we study the first two in detail.

It turns out that the epidemic occurs if $\frac{dI}{dt} > 0$, it doesn't if $\frac{dI}{dt} < 0$, so for the epidemic to occur we have to have $\propto S > \beta$ implying

$S > \frac{\beta}{\propto}$. For the epidemic to terminate the rate of change of I has to be negative this implies that $S < \frac{\beta}{\propto}$.

Definition

(Basic Reproductive Number).

The basic reproductive number R_0 (the average number of persons infected one case in a totally susceptible population in absence of interventions aimed at controlling the infection). Since $S = 1$ initially the ratio $\propto \frac{S}{\beta} = \frac{\propto}{\beta} = R_0$

This is one of the most important in the SIR modeling of any epidemic R_0 is especially important in this case as it will inform one as to when an epidemic is in progress so if $R_0 > 1$ an epidemic will occur and if $R_0 < 1$ there will be no epidemic. The values of R_0 are known for various diseases.

$$I(S) = -S + \frac{1}{R_0} Lns + 1 \text{——} E.q14$$

Herd Immunity:

For this portion of the model we use P to be the proportion of susceptible population that is immunized before the outbreak of an epidemic and assume the above mentioned conditions new equations governing the disease can be written as

$$\grave{S} = \propto (1 - P)SI$$

$$\grave{I} = \propto (1 - P)SI - \beta I \text{——} E.q\ 15$$

An outbreak of the epidemic mathematically means that

$$\grave{I} > 0 \Rightarrow \propto (1-P)SI - \beta I > 0$$

$$\Rightarrow \propto (1-P)S > \beta \Rightarrow (\propto/\beta)(1-P)S > 1 \Rightarrow R_0 > \frac{1}{1-P} \text{E.q16}$$

4. CONCLUSION

In this brief presentation after an overview of differential equations with the general solution

$$b_n(x)y^n + b_{n-1}(x)y^{(n-1)} + \ldots + b_2(x)y^{\backslash\backslash} + b_1(x)y^{\backslash} + b_0(x)y \quad = g(x)$$

and the linear differential equation system.

$$\dot{x}(t) = A(t)X(t) + b(t)$$

and display the dynamic system and solve it

$$\acute{x}(t) = f(x(t), u(t), t)$$

$$y(t) = g(x(t), u(t), t)$$

It is the application of predator- prey to the ecosystem

$$\frac{dx}{dt} = ax - bxy - mx^2$$

$$\frac{dy}{dt} = cxy - ey - ny^2$$

It was concluded that partial differential equations play role and contribute to solution of these system .the aim of the study and the paper is to address the importance of partial differential equations.

REFERENCES

[1] James C Robinson ,an introduction to ordinary differential equations , Cambridge university press 2004

[2] Prof. Nassar Hassan Abdel, Aal - Ordinary Differential Equations. Riyadh 1434 - Al Rashed Library Buraidah.

[3] Richard Bronson, PhD Seton hall University Copyright © 2006, 1994, 1973 by The McGraw-Hill Companies

[4] Gabriel Nagy, Ordinary Differential Equations, Michigan state university – august 15, 2016

[5] Emmanuel Hainry, Reachability in linear dynamical systems - LORIA, University Henri Poincar- Emmanuel. Hainry @loria.fr

[6] ClaudioMelchiorri – Stability Analysis of dynamic systems – Dipartimento di Ingegneriadell’EnergiaElettrica e dell’Informazione (DEI) Università di Bologna, Email: claudio.melchiorri@unibo.it

[7] Courtney Brown- Differential Equations A Modeling Approach - Emory University- Copyright 2007_ by Sage Publications, Inc.

[8] Ross Beckley - Modeling epidemics with differential equations -Tennessee State University, 2Philander Smith College. June 21, 2013 Abstract.